MODIFIED RATIO-CUM-PRODUCT ESTIMATORS OF POPULATION MEAN IN LINEAR SYTEMATIC SAMPLING UNDER TWO-PHASE SAMPLING SCHEME

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Abstract: This paper is basically aimed at suggesting two modified ratio-cum-product estimators having two auxiliary variables, under the linear systematic sampling. These suggested ratio-cum-product modified estimators under the two-phase sampling scheme were suggested using linear transformation technique, the biases and mean squared errors (MSEs) of the corresponding modified estimators under cases I and II were derived and established (where Case I refers to the case whereby the second sample S_2 is drawn from the first sample S_1 and Case II refers to the case whereby the second sample S_2 is drawn from the main population under study), the efficiency conditions under which the two modified estimators would be more efficient than relative existing ones were derived and established and the relative efficiency of these modified estimators were empirically determined and established using five real life data sets which were obtained from various sources. From the results of the empirical study using real life data sets, it was concluded that the suggested modified estimators in this study demonstrated high relative efficiency over existing related estimators.

Keywords: Efficiency, finite population, mean squared error, ratio estimator, variance

Introduction

The main aim of survey statisticians is to reduce errors either by devising suitable sampling schemes or by formulating efficient estimators of the parameters (Singh and Solanki, 2013). To reduce the errors, various researchers have attempted to use additional information, which is correlated to the information under the study and about which the information is available before commencing the survey, this information is referred to as the auxiliary information. In survey of finite populations, systematic sampling is the most commonly used probability design due to its simplicity (Madow and Madow, 1944). Besides its simplicity, systematic sampling provides more efficient estimators than that of simple random sampling nor stratified random sampling for certain types of population (Cochran, 1946; Hajeck, 1959). It should be noted that there is possibility of variations; which initiates the emphases laid on the need of efficiency of desired estimators which can be achieved by the judicious use of auxiliary information when there exist a strong correlation between this auxiliary variable and the variable under study, either positive or negative. At times, when generating estimates of the population parameters of the variable under study, the population parameters of the auxiliary variable may not be known forehand, hence, the initiation of two–phase (double) sampling in some cases which happens to be cost effective for generating a more reliable estimate than that of single–phase sampling.

The present study focuses on the application of root transformation on auxiliary variables and use of conventional ratio–cum–product estimators in improving the efficiency of Tailor *et al.* (2013) on the ratio–cum–product estimators of finite population mean and the empirical study on the efficiency of the modified estimators is limited to use of real life data.

The work of Swain (1967) was broadened by Tailor *et al*. (2013) by integrating the information of a second auxiliary variable Z. The estimator derived in his work is called the ratio–cum–product estimator. Hence, the derived estimator and its corresponding properties are mathematically noted as;

$$\hat{\theta}_5 = \bar{y}_{sys}\left(\frac{\bar{X}}{\bar{x}_{sys}}\right)\left(\frac{\bar{z}_{sys}}{\bar{Z}}\right) \quad (1.1)$$

$$MSE\left(\hat{\theta}_5\right) = \left(\frac{1}{n}-\frac{1}{N}\right)\bar{Y}^2\Big(\rho_y^* C_y^2 + \rho_x^* C_x^2 - 2\rho_{yx}C_yC_x\sqrt{\rho_y^*\rho_x^*} + \rho_z^* C_z^2 - 2\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*} + 2\rho_{yz}C_yC_z\sqrt{\rho_y^*\rho_z^*}\Big) \quad (1.2)$$

Khan and Singh (2015) suggested a modified estimator for the population mean under systematic sampling by modifying the work of Tailor *et al*. (2013) using linear and power transformation techniques. Following this, the modified estimator suggested and its resulting mean squared error are mathematically noted as;

$$\theta_6 = \bar{y}_{sys}\left(\frac{\bar{X}}{\bar{X}+b_{yx}\left(\bar{x}_{sys}-\bar{X}\right)}\right)^{\delta_1}\left(\frac{\bar{Z}+b_{yz}\left(\bar{z}_{sys}-\bar{Z}\right)}{\bar{Z}}\right)^{\delta_2} \quad (1.3)$$

$$MSE(\theta_6) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}^2 \rho_y^* \left(C_y^2 - k^{*2} C_z^2 - \frac{\left(k^* k^{**} C_z^2 - k C_x^2\right)^2}{\left(C_x^2 - k^{**2} C_z^2\right)} \right) \quad (1.4)$$

where $k = \frac{\rho_{yx} C_y}{C_x},\ k^* = \frac{\rho_{yz} C_y}{C_z}, k^{**} = \frac{\rho_{xz} C_x}{C_z}$

The work of Tailor *et al*. (2013) was transformed by Khan (2016) using the exponential function hence, suggested a newly modified estimator for the population mean under systematic sampling. Following this, the modified estimator suggested and its resulting mean squared error are mathematically noted as;

$$\theta_7 = \bar{y}_{sys} \exp\left(\frac{f\left(\bar{X} - \bar{x}_{sys}\right)}{\bar{X} + (g-1)\bar{x}_{sys}} \right) \exp\left(\frac{h\left(\bar{Z} - \bar{z}_{sys}\right)}{\bar{Z} + (\eta - 1)\bar{z}_{sys}} \right) \quad (1.5)$$

where $-\infty < f < \infty,\ -\infty < h < \infty,\ g > 0,\ \eta > 0$

$$MSE(\theta_7) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}^2 \rho_y^* \left(C_y^2 + \frac{C_x^2}{\delta_2^2}\left(\left(\delta_1^2 + \delta_3^2 C_x^2 C_z^2 + 2k^{**} C_z^2 \delta_1 \delta_3\right)\right) - 2\delta_2\left(k\delta_1 + k^* C_z^2 \delta_3\right)\right) \quad (1.6)$$

where $\delta_1 = kC_x^2 - k^* k^{**} C_z^2,\ \delta_2 = C_x^2 - k^{**2} C_z^2,\ \delta_3 = k^* - kk^{**},$

The procedure for two–phase–sampling involves the technique of the simple random sampling without replacement (SRSWOR) which has to be implemented initially to obtain a first sample of size *n′* and this first sample is basically used to study the auxiliary information X only. The next – phase is to select a second sample of size *n* and this second sample is used to study both the auxiliary variable X and the variable of interest Y simultaneously. This second–phase–sampling could be done in two ways:

First Case: $S_2 \subset S_1$ (the second sample S_2 is drawn from the first sample S_1)

Second Case: $S_2 \subset \Omega_N$ (the second sample S_2 is drawn from the main population under study)

Taking into consideration the selected samples S_1 and S_2 above, the sample means of the auxiliary variable and the study variable are represented mathematically as:

$$\bar{y}_{sys} = \frac{1}{n}\sum_{i \in S_2} y_i,\ \bar{x}_{sys} = \frac{1}{n}\sum_{i \in S_2} x_i,\ \bar{z}_{sys} = \frac{1}{n}\sum_{i \in S_2} z_i,\ \bar{x}_{sys}^{'} = \frac{1}{n^{'}}\sum_{i \in S_1} x_i \text{ and } \bar{z}_{sys}^{'} = \frac{1}{n^{'}}\sum_{i \in S_1} z_i\ .$$

Hence, the conventional ratio and product estimators for the two – phase estimation are mathematically noted as;

$$\hat{\theta}_1^{(d)} = \bar{y}_{sys}\left(\frac{\bar{x}_{sys}^{'}}{\bar{x}_{sys}}\right) \quad (1.7)$$

$$\hat{\theta}_2^{(d)} = \bar{y}_{sys}\left(\frac{\bar{x}_{sys}}{\bar{x}_{sys}^{'}}\right) \quad (1.8)$$

Thus, their resulting bias and MSE up to their first order of approximation are denoted respectively as:

$$Bias\left(\hat{\theta}_1^{(d)}\right)_I = -\bar{Y}\left(\theta_3 \rho_x^* C_x^2 + \frac{1}{2}\theta_3 \rho_{yx} C_y C_x \sqrt{\rho_y^* \rho_x^*}\right) \quad (1.9)$$

$$Bias\left(\hat{\theta}_1^{(d)}\right)_{II} = \bar{Y}\left(\theta_1 \rho_x^* C_x^2 - \theta_2 \rho_{yx} C_y C_x \sqrt{\rho_y^* \rho_x^*}\right) \quad (1.10)$$

$$Bias\left(\hat{\theta}_2^{(d)}\right)_I = \bar{Y}\theta_3 \rho_{yx} C_y C_x \sqrt{\rho_y^* \rho_x^*} \quad (1.11)$$

$$Bias\left(\hat{\theta}_2^{(d)}\right)_{II} = \bar{Y}\left(\theta_1 \rho_x^* C_x^2 + \theta_2 \rho_{yx} C_y C_x \sqrt{\rho_y^* \rho_x^*}\right) \quad (1.12)$$

$$MSE\left(\hat{\theta}_1^{(d)}\right)_I = \bar{Y}^2\left(\theta_2 \rho_y^* C_y^2 + \theta_3 \rho_x^* C_x^2 - 2\theta_3 \rho_{yx} C_y C_x \sqrt{\rho_y^* \rho_x^*}\right) \quad (1.13)$$

$$MSE\left(\hat{\theta}_1^{(d)}\right)_{II} = \bar{Y}^2\left(\theta_2 \rho_y^* C_y^2 + \theta_3 \rho_x^* C_x^2 - 2\theta_2 \rho_{yx} C_y C_x \sqrt{\rho_y^* \rho_x^*}\right) \quad (1.14)$$

$$MSE\left(\hat{\theta}_2^{(d)}\right)_I = \bar{Y}^2\left(\theta_2 \rho_y^* C_y^2 + \theta_3 \rho_x^* C_x^2 + 2\theta_3 \rho_{yx} C_y C_x \sqrt{\rho_y^* \rho_x^*}\right) \quad (1.15)$$

$$MSE\left(\hat{\theta}_2^{(d)}\right)_{II} = \bar{Y}^2\left(\theta_2 \rho_y^* C_y^2 + \theta_3 \rho_x^* C_x^2 + 2\theta_2 \rho_{yx} C_y C_x \sqrt{\rho_y^* \rho_x^*}\right) \quad (1.16)$$

where $\rho_x^* = 1 + (n-1)\rho_x$

The exponential – type of ratio and product estimators under systematic two-phase sampling were suggested by Singh *et al*. (2011). These suggested estimators are modified editions of the initially suggested modified estimators by Singh and Vishwakarma (2007) for the simple random sampling technique. Hence, the estimators suggested and their resulting bias and mean squared errors (MSEs) are mathematically noted as;

$$\hat{\theta}_3^{(d)} = \bar{y}_{sys} \exp\left(\frac{\bar{x}'_{sys} - \bar{x}_{sys}}{\bar{x}'_{sys} + \bar{x}_{sys}}\right) \tag{1.17}$$

$$\hat{\theta}_4^{(d)} = \bar{y}_{sys} \exp\left(\frac{\bar{x}_{sys} - \bar{x}'_{sys}}{\bar{x}_{sys} + \bar{x}'_{sys}}\right) \tag{1.18}$$

$$Bias\left(\hat{\theta}_3^{(d)}\right)_I = \bar{Y}\left(\frac{3}{8}\theta_2\rho_x^* C_x^2 - \frac{5}{8}\theta_1\rho_x^* C_x^2 - \frac{1}{2}\theta_3\rho_{yx}C_y C_x\sqrt{\rho_y^*\rho_x^*}\right) \tag{1.19}$$

$$Bias\left(\hat{\theta}_3^{(d)}\right)_{II} = \bar{Y}\left(\frac{1}{8}\theta_1\rho_x^* C_x^2 + \frac{3}{8}\theta_2\rho_x^* C_x^2 - \frac{1}{2}\theta_2\rho_{yx}C_y C_x\sqrt{\rho_y^*\rho_x^*}\right) \tag{1.20}$$

$$Bias\left(\hat{\theta}_4^{(d)}\right)_I = \bar{Y}\left(-\frac{1}{8}\theta_3\rho_x^* C_x^2 + \frac{1}{2}\theta_3\rho_{yx}C_y C_x\sqrt{\rho_y^*\rho_x^*}\right) \tag{1.21}$$

$$Bias\left(\hat{\theta}_4^{(d)}\right)_{II} = \bar{Y}\left(-\frac{1}{8}\theta_2\rho_x^* C_x^2 + \frac{3}{8}\theta_1\rho_x^* C_x^2 + \frac{1}{2}\theta_2\rho_{yx}C_y C_x\sqrt{\rho_y^*\rho_x^*}\right) \tag{1.22}$$

$$MSE\left(\hat{\theta}_3^{(d)}\right)_I = \bar{Y}^2\left(\theta_2\rho_y^* C_y^2 + \frac{1}{4}\theta_3\rho_x^* C_x^2 - \theta_3\rho_{yx}C_y C_x\sqrt{\rho_y^*\rho_x^*}\right) \tag{1.23}$$

$$MSE\left(\hat{\theta}_3^{(d)}\right)_{II} = \bar{Y}^2\left(\theta_2\rho_y^* C_y^2 + \frac{1}{4}\theta_3\rho_x^* C_x^2 - \theta_2\rho_{yx}C_y C_x\sqrt{\rho_y^*\rho_x^*}\right) \tag{1.24}$$

$$MSE\left(\hat{\theta}_4^{(d)}\right)_I = \bar{Y}^2\left(\theta_2\rho_y^* C_y^2 + \frac{1}{4}\theta_3\rho_x^* C_x^2 + \theta_3\rho_{yx}C_y C_x\sqrt{\rho_y^*\rho_x^*}\right) \tag{1.25}$$

$$MSE\left(\hat{\theta}_4^{(d)}\right)_{II} = \bar{Y}^2\left(\theta_2\rho_y^* C_y^2 + \frac{1}{4}\theta_3\rho_x^* C_x^2 + \theta_2\rho_{yx}C_y C_x\sqrt{\rho_y^*\rho_x^*}\right) \tag{1.26}$$

Suggested estimators

Having studied the work of Khan and Singh (2015) and Khan (2016) critically and also motivated by the work of Singh *et al*. (2011) and Singh (2015), the modified suggested estimators under the two – phased systematic sampling scheme are mathematically noted as follows;

$$\bar{Z}_{sys} = \frac{\bar{y}_{sys}}{\bar{x}_{sys}}\left(\bar{x}'_{sys} + n'b_{xz}\left(\bar{z}'_{sys} - \bar{z}_{sys}\right)\right) \tag{2.1}$$

$$\bar{Z}'_{sys} = \bar{y}_{sys} \exp\left(\frac{\bar{x}'_{sys} + n'b_{xz}\left(\bar{z}'_{sys} - \bar{z}_{sys}\right) - \bar{x}_{sys}}{\bar{x}'_{sys} + n'b_{xz}\left(\bar{z}'_{sys} - \bar{z}_{sys}\right) + \bar{x}_{sys}}\right) \tag{2.2}$$

Properties (bias and MSES) for the modified estimators

The Taylor's series expansion approach was applied in this section for the derivation of the bias and MSEs of the modified estimators up to the first order (second degree) approximation.

For the derivation of these properties (bias and MSEs), the following error terms e_0, e_1, e_2, e_3 and e_4 are assumed as;

$$e_0 = \frac{\bar{y}_{sys} - \bar{Y}}{\bar{Y}},\ e_1 = \frac{\bar{x}_{sys} - \bar{X}}{\bar{X}},\ e_2 = \frac{\bar{x}'_{sys} - \bar{X}}{\bar{X}},\ e_3 = \frac{\bar{z}_{sys} - \bar{Z}}{\bar{Z}} \text{ And } e_4 = \frac{\bar{z}'_{sys} - \bar{Z}}{\bar{Z}}$$

Such that; $|e_i| \approx 0,\ i = 0, 1, 2, 3, 4$, and the resulting expectation of these error terms under cases I and II are derived as follows;

For case I:

$$
\left.
\begin{array}{l}
E(e_0)=E(e_1)=E(e_2)=0,\ E(e_0^2)=\theta_2\rho_y^*C_y^2,\ E(e_1^2)=\theta_2\rho_x^*C_x^2, E(e_2^2)=\theta_1\rho_x^*C_x^2 \\
E(e_3^2)=\theta_2\rho_z^*C_z^2,\ E(e_4^2)=\theta_2\rho_z^*C_z^2,\ E(e_0e_1)=\theta_2\rho_{yx}C_yC_x\sqrt{\rho_y^*\rho_x^*}, \\
E(e_0e_2)=\theta_1\rho_{yx}C_yC_x\sqrt{\rho_y^*\rho_x^*}\ , E(e_0e_3)=\theta_2\rho_{yz}C_yC_z\sqrt{\rho_y^*\rho_z^*},\ E(e_0e_4)=\theta_1\rho_{yz}C_yC_z\sqrt{\rho_y^*\rho_z^*}, \\
E(e_1e_2)=\theta_1\rho_x^*C_x^2, E(e_1e_3)=\theta_2\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*},\ E(e_1e_4)=\theta_2\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*}, \\
E(e_2e_3)=\theta_1\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*}E(e_3e_4)=\theta_1\rho_z^*C_z^2, E(e_2e_4)=\theta_1\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*},\ \theta_1=\dfrac{1}{n'}-\dfrac{1}{N}, \\
\theta_2=\dfrac{1}{n}-\dfrac{1}{N},\ \theta_3=\theta_2-\theta_1,\ \theta_4=\theta_2+\theta_1, \rho_y^*=1+(n-1)\rho_y,\quad \rho_x^*=1+(n-1)\rho_x, \\
\rho_z^*=1+(n-1)\rho_z,\ \phi=(1-f)/n,\ f=n/N
\end{array}
\right\} \quad (3.1)
$$

For case II

$$
\left.
\begin{array}{l}
E(e_0)=E(e_1)=E(e_2)=0,\ E(e_0^2)=\theta_2\rho_y^*C_y^2,\ E(e_1^2)=\theta_2\rho_x^*C_x^2, E(e_2^2)=\theta_1\rho_x^*C_x^2 \\
E(e_3^2)=\theta_2\rho_z^*C_z^2,\ E(e_4^2)=\theta_2\rho_z^*C_z^2,\ E(e_0e_1)=\theta_2\rho_{yx}C_yC_x\sqrt{\rho_y^*\rho_x^*}, \\
E(e_0e_3)=\theta_2\rho_{yz}C_yC_z\sqrt{\rho_y^*\rho_z^*}, E(e_1e_3)=\theta_2\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*}, E(e_2e_4)=\theta_1\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*} \\
E(e_1e_4)=0,\ E(e_2e_3)=0, E(e_0e_4)=0,\ E(e_1e_2)=0,\ E(e_0e_2)=0 \\
\theta_1=\dfrac{1}{n'}-\dfrac{1}{N}, \theta_2=\dfrac{1}{n}-\dfrac{1}{N},\ \theta_3=\theta_2-\theta_1,\ \theta_4=\theta_2+\theta_1 \\
\rho_y^*=1+(n-1)\rho_y,\quad \rho_x^*=1+(n-1)\rho_x,\quad \rho_z^*=1+(n-1)\rho_z,\ \phi=(1-f)/n,\ f=n/N
\end{array}
\right\} \quad (3.2)
$$

Bias and MSE of $\bar{Z}_{sys}$

Expressing the assumed error terms e_0, e_1, e_2, e_3 and e_4 into (2.1):

$$\bar{Z}_{sys}=\frac{\bar{Y}}{\bar{X}}(1+e_0)(1+e_1)^{-1}\left[\bar{X}+\bar{X}e_2+b_{xz}(n')\left[\bar{Z}e_4-\bar{Z}e_3\right]\right] \quad (3.3)$$

Simplifying to first order approximation (3.3):

$$\bar{Z}_{sys}=R\left(1-e_1+e_1^2+e_0-e_0e_1\right)\left[\bar{X}+\bar{X}e_2+b_{xz}(n')\bar{Z}e_4-b_{xz}(n')\bar{Z}e_3\right] \quad (3.4)$$

where $R=\dfrac{\bar{Y}}{\bar{X}}$

Further simplifying (3.4):

$$\bar{Z}_{sys}=\bar{Y}\left[1+e_2-e_1-e_0e_2+e_1^2+e_0+e_0e_2-e_0e_1\right]+Rb_{xz}n'\bar{Z}\left[e_4-e_3-e_1e_4+e_1e_3+e_0e_4-e_0e_3\right] \quad (3.5)$$

Now, subtracting $\bar{Y}$ from both sides of (3.5):

$$\bar{Z}_{sys}-\bar{Y}=\bar{Y}\left[e_2-e_1+e_1^2+e_0-e_0e_1\right]+Rb_{xz}n'\bar{Z}\left[e_4-e_3-e_1e_4+e_1e_3+e_0e_4-e_0e_3\right] \quad (3.6)$$

Taking the expectation of (3.6) and applying the results from (3.1), the bias of the modified estimator $\bar{Z}_{sys}$ under case I is derived to be:

$$
\begin{aligned}
Bias\left(\bar{Z}_{sys}\right)_I=\bar{Y}\Big(&\theta_2C_x^2\rho_x^*-\theta_2\rho_{yx}C_yC_x\sqrt{\rho_y^*\rho_x^*} \\
&+Rb_{xz}n'\bar{Z}\left[\theta_3\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*}-\theta_3\rho_{yz}C_yC_z\sqrt{\rho_y^*\rho_z^*}\right]\Big)
\end{aligned} \quad (3.7)
$$

Also taking the expectation of (3.6) then applying the results from (3.2), the bias of the modified estimator $\bar{Z}_{sys}$ under case II is derived to be:

$$Bias\left(\bar{Z}_{sys}\right)_{II}=\bar{Y}\left(\theta_2 C_x^2\rho_x^*-\theta_2\rho_{yx}C_yC_x\sqrt{\rho_y^*\rho_x^*}\right.$$
$$\left.+Rb_{xz}n'\bar{Z}\left[\theta_2\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*}-\theta_2\rho_{yz}C_yC_z\sqrt{\rho_y^*\rho_z^*}\right]\right) \quad (3.8)$$

Taking the square of both sides of (3.6), the expectation of the resulting equation and applying the results from (3.1), the MSE of the modified estimator $\bar{Z}_{sys}$ to its first order approximation under case I is derived to be;

$$MSE(\bar{Z}_{sys})_I=\bar{Y}^2\left(\theta_2C_y^2\rho_y^*+\theta_3C_x^2\rho_x^*-2\theta_3\rho_{yx}C_yC_x\sqrt{\rho_y^*\rho_x^*}\right)+R^2b_{xz}^2n'^2\bar{Z}^2\theta_2C_z^2\rho_z^*$$
$$-2\theta_3\bar{Y}\bar{Z}Rb_{xz}n'\left(\rho_{yz}C_yC_z\sqrt{\rho_y^*\rho_z^*}-\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*}\right) \quad (3.9)$$

Taking the square of both sides of (3.6), the expectation of the resulting equation and applying the results from (3.2), the MSE of the modified estimator $\bar{Z}_{sys}$ to its first order approximation under case II is derived to be;

$$MSE\left(\bar{Z}_{sys}\right)_{II}=\bar{Y}^2\left[\theta_2C_y^2\rho_y^*+\theta_3C_x^2\rho_x^*-2\theta_2\rho_{yx}C_yC_x\sqrt{\rho_y^*\rho_x^*}\right]$$
$$+R^2b_{xz}^2n'^2\bar{Z}^2\theta_4C_z^2\rho_z^*+2\bar{Y}\bar{Z}Rb_{xz}n'\left[-\theta_2\rho_{yz}C_yC_z\sqrt{\rho_y^*\rho_z^*}+\theta_4\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*}\right] \quad (3.10)$$

Bias and MSE of $\bar{Z}'_{sys}$

Expressing the assumed error terms e_0, e_1, e_2, e_3 and e_4 into (2.2):

$$\bar{Z}'_{sys}=(1+e_0)\bar{Y}\exp\left[\frac{\bar{X}e_2-\bar{X}e_1+b_{xz}(n')\left[\bar{Z}e_4-\bar{Z}e_3\right]}{2\bar{X}+\bar{X}e_2+\bar{X}e_1+b_{xz}(n')\left[\bar{Z}e_4-\bar{Z}e_3\right]}\right] \quad (3.11)$$

Simplifying to first order approximation (3.11):

$$\bar{Z}'_{sys}-\bar{Y}=\bar{Y}\left(\frac{e_2}{2}-\frac{e_1}{2}+\frac{b_{xz}n'\bar{Z}}{2\bar{X}}(e_4-e_3)-\frac{e_2^2}{4}+\frac{3}{8}e_1^2-\frac{1}{8}\frac{b_{xz}^2n'^2\bar{Z}^2}{\bar{X}^2}(e_4^2+e_3^2-2e_3e_4)-\frac{e_1e_2}{2}\right.$$
$$e_2-\frac{e_1}{2}+\frac{b_{xz}n'\bar{Z}}{2\bar{X}}(e_4-e_3)-\frac{e_2^2}{4}+\frac{3}{8}e_1^2-\frac{1}{8}\frac{b_{xz}^2n'^2\bar{Z}^2}{\bar{X}^2}(e_4^2+e_3^2-2e_3e_4)-\frac{e_1e_2}{2} \quad (3.12)$$
$$\left.+\frac{b_{xz}n'\bar{Z}}{2\bar{X}}(e_1e_4-e_1e_3)+e_0+e_0e_2-\frac{e_0e_1}{2}+\frac{b_{xz}n'\bar{Z}}{4\bar{X}}(e_0e_4-e_0e_3)\right)$$

Taking the expectation of (3.12) and applying the results from (3.1), the bias of the modified estimator $\bar{Z}'_{sys}$ under case I is derived to be:

$$Bias\left(\bar{Z}'_{sys}\right)_I=\bar{Y}\left(\frac{3}{4}\theta_1C_x^2\rho_x^*(\frac{1}{2}\theta_2-\theta_1)-\frac{1}{8}\frac{b_{xz}^2n'^2\bar{Z}^2}{\bar{X}}\theta_3C_z^2\rho_z^*\right.$$
$$\left.-\theta_3\frac{b_{xz}n'\bar{Z}}{4\bar{X}}\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*}-\theta_3\rho_{yx}C_yC_x\sqrt{\rho_y^*\rho_x^*}-\theta_3\frac{b_{yz}n'\bar{Z}}{2\bar{X}}\rho_{yz}C_yC_z\sqrt{\rho_y^*\rho_z^*}\right) \quad (3.13)$$

Also taking the expectation of (3.12) and applying the results from (3.2), the bias of the modified estimator $\bar{Z}'_{sys}$ under case II is derived to be:

$$Bias\left(\bar{Z}'_{sys}\right)_{II}=\bar{Y}\left(-\frac{1}{4}\theta_1C_x^2\rho_x^*+\frac{3}{8}\theta_2C_x^2\rho_x^*-\frac{1}{8}\frac{b_{xz}^2n'^2\bar{Z}^2}{\bar{X}}(\theta_4C_z^2\rho_z^*)\right.$$
$$\left.-\frac{b_{xz}n'\bar{Z}}{4\bar{X}}\theta_2\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*}+\theta_2\rho_{yx}C_yC_x\sqrt{\rho_y^*\rho_x^*}-\frac{b_{yz}n'\bar{Z}}{2\bar{X}}\theta_2\rho_{yz}C_yC_z\sqrt{\rho_y^*\rho_z^*}\right) \quad (3.14)$$

Taking the square of sides of (3.12), the expectation of the resulting equation and applying the results from (3.1), the MSE of the modified estimator $\bar{Z}'_{sys}$ to its first order approximation under case I is obtained to be:

$$MSE\left(\bar{Z}'_{sys}\right)_I = \bar{Y}^2\left(\theta_2 C_y^2\rho_y^* + \frac{\theta_3}{4}C_x^2\rho_x^* + \frac{b_{xy}^2 n'\bar{Z}^2}{4\bar{X}^2}\theta_3 C_z^2\rho_z^* + \theta_3\rho_{yx}C_xC_y\sqrt{\rho_x^*\rho_y^*} - \frac{b_{xy}n'\bar{Z}}{\bar{X}}\theta_3\rho_{yz}C_zC_y\sqrt{\rho_y^*\rho_z^*} + \frac{b_{xy}n'\bar{Z}}{2\bar{X}}\theta_3\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*}\right) \tag{3.15}$$

Taking the square of both sides of (3.12), the expectation of the resulting equation and applying the results from (3.2), the MSE of the modified estimator $\bar{Z}'_{sys}$ to its first order approximation under case II is derived to be:

$$MSE\left(\bar{Z}'_{sys}\right)_{II} = \bar{Y}^2\left(\theta_2 C_y^2\rho_y^* + \frac{\theta_4}{4}C_x^2\rho_x^* + \frac{b_{xy}^2 n'\bar{Z}^2}{4\bar{X}^2}\theta_4 C_z^2\rho_z^* - \theta_2\rho_{yx}C_xC_y\sqrt{\rho_x^*\rho_y^*} - \frac{b_{xy}n'\bar{Z}}{\bar{X}}\theta_2\rho_{yz}C_zC_y\sqrt{\rho_y^*\rho_z^*} + \frac{b_{xy}n'\bar{Z}}{2\bar{X}}\theta_1 C_x^2\rho_x^* + \frac{b_{xy}n'\bar{Z}}{2\bar{X}}\theta_2\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*}\right) \tag{3.16}$$

Theoretical efficiency comparisons

In this section, MSEs of the newly modified estimators are compared to the MSEs of some existing related estimators, hence, establishing the efficiency conditions of the newly modified estimators over some existing ones.

Efficiency condition of first estimator $\bar{Z}_{sys}$ over some related estimators under Case I

i. Comparing Sample mean $\bar{y}_{sys}$ with $\bar{Z}_{sys}$

$$\operatorname{var}\left(\bar{y}_{sys}\right) - MSE\left(\bar{Z}_{sys}\right)_I > 0 \tag{4.1}$$

$$\rho_{yx} < \frac{\bar{Y}^2\left(\theta_3 C_x^2\rho_x^* - 2\theta_3\rho_{yx}C_yC_x\sqrt{\rho_y^*\rho_x^*}\right) + R^2 b_{xz}^2 n'^2\bar{Z}^2\theta_2 C_z^2\rho_z^* - 2\theta_3\bar{Y}\bar{Z}Rb_{xz}n'\left(\rho_{yz}C_yC_z\sqrt{\rho_y^*\rho_z^*} - \rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*}\right)}{2\bar{Y}^2\theta_3 C_yC_x\sqrt{\rho_y^*\rho_x^*}} \tag{4.2}$$

Estimator $\bar{Z}_{sys}$ is more efficient than $\bar{y}_{sys}$ when (4.2) is satisfied.

ii. Comparing Ratio estimator $\hat{\theta}_1^{(d)}$ with $\bar{Z}_{sys}$

$$MSE\left(\hat{\theta}_1^{(d)}\right) - MSE\left(\bar{Z}_{sys}\right)_I > 0 \tag{4.3}$$

$$\rho_{xz} < \frac{-b_{xz}n'\bar{Z}\theta_2 C_z\sqrt{\rho_z^*} + 2\bar{X}\theta_3\rho_{yz}C_y\sqrt{\rho_y^*}}{2\theta_3\bar{X}C_x\sqrt{\rho_x^*}} \tag{4.4}$$

Estimator $\bar{Z}_{sys}$ is more efficient than $\hat{\theta}_1^{(d)}$ when (4.4) is satisfied.

iii. Comparing Product estimator $\hat{\theta}_2^{(d)}$ with $\bar{Z}_{sys}$

$$MSE\left(\hat{\theta}_2^{(d)}\right) - MSE\left(\bar{Z}_{sys}\right)_I > 0 \tag{4.5}$$

$$\rho_{yx} < \frac{R^2 b_{xz}^2 n'^2\bar{Z}^2\theta_2 C_z^2\rho_z^* + 2\bar{Y}\bar{Z}Rb_{xz}n'\left[-\theta_3\rho_{yz}C_yC_z\sqrt{\rho_y^*\rho_z^*} + \theta_3\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*}\right]}{4\bar{Y}^2\theta_3 C_yC_x\sqrt{\rho_y^*\rho_x^*}} \tag{4.6}$$

Estimator $\bar{Z}_{sys}$ is more efficient than $\hat{\theta}_2^{(d)}$ when (4.6) is satisfied.

iv. Comparing Singh *et al*. (2011) Exponential ratio/product estimator $\hat{\theta}_3^{(d)}$ with $\bar{Z}_{sys}$

$$MSE\left(\hat{\theta}_3^{(d)}\right) - MSE\left(\bar{Z}_{sys}\right)_I > 0 \tag{4.7}$$

$$\rho_{yx} < \frac{2\overline{Y}\overline{Z}Rb_{xz}n'\left[-\theta_3\rho_{yz}C_yC_z\sqrt{\rho_y^*\rho_z^*}+\theta_3\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*}\right]-\frac{3}{4}\theta_3\overline{Y}^2C_x^2\rho_x^* +R^2b_{xz}^2n'^2\overline{Z}^2\theta_2C_z^2\rho_z^*}{\theta_3\overline{Y}^2C_yC_x\sqrt{\rho_x^*\rho_y^*}} \quad (4.8)$$

Estimator $\overline{Z}_{sys}$ is more efficient than $\hat{\theta}_3^{(d)}$ when (4.8) is satisfied.

v. Comparing Singh *et al*. (2011) Exponential ratio/product estimator $\hat{\theta}_4^{(d)}$ with $\overline{Z}_{sys}$

$$MSE\left(\hat{\theta}_4^{(d)}\right)-MSE\left(\overline{Z}_{sys}\right)_I > 0 \quad (4.9)$$

$$\rho_{yx} > \frac{2\overline{Y}\overline{Z}Rb_{xz}n'\left[-\theta_3\rho_{yz}C_yC_z\sqrt{\rho_y^*\rho_z^*}+\theta_3\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*}\right]-\frac{3}{4}\theta_3\overline{Y}^2C_x^2\rho_x^* +R^2b_{xz}^2n'^2\overline{Z}^2\theta_2C_z^2\rho_z^*}{3\theta_3\overline{Y}^2C_yC_x\sqrt{\rho_x^*\rho_y^*}} \quad (4.10)$$

Estimator $\overline{Z}_{sys}$ is more efficient than $\hat{\theta}_4^{(d)}$ when (4.10) is satisfied.

Efficiency condition of $\overline{Z}_{sys}$ over some related estimators under Case II

i. Comparing Sample mean $\overline{y}_{sys}$ with $\overline{Z}_{sys}$

$$\operatorname{var}\left(\overline{y}_{sys}\right)-MSE\left(\overline{Z}_{sys}\right)_{II} > 0 \quad (4.11)$$

$$\rho_{yx} < \frac{2\overline{Y}\overline{Z}Rb_{xz}n'\left[-\theta_2\rho_{yz}C_yC_z\sqrt{\rho_y^*\rho_z^*}+\theta_4\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*}\right]+ \overline{Y}^2\theta_3C_x^2\rho_x^*+R^2b_{xz}^2n'^2\overline{Z}^2\theta_4C_z^2\rho_z^*}{2\theta_2C_yC_x\sqrt{\rho_y^*\rho_x^*}} \quad (4.12)$$

Estimator $\overline{Z}_{sys}$ is more efficient than $\overline{y}_{sys}$ when (4.12) is satisfied.

ii. Comparing Ratio estimator $\hat{\theta}_1^{(d)}$ with $\overline{Z}_{sys}$

$$MSE\left(\hat{\theta}_1^{(d)}\right)_{II}-MSE\left(\overline{Z}_{sys}\right)_{II} > 0 \quad (4.13)$$

$$\rho_{xz} > \frac{b_{xz}n'\overline{Z}\theta_2C_z\sqrt{\rho_z^*}-2\overline{X}\theta_3\rho_{yz}C_y\sqrt{\rho_y^*}}{2\theta_3\overline{X}C_x\sqrt{\rho_x^*}} \quad (4.14)$$

Estimator $\overline{Z}_{sys}$ is more efficient than $\hat{\theta}_1^{(d)}$ when (4.14) is satisfied.

iii. Comparing product estimator $\hat{\theta}_2^{(d)}$ with $\overline{Z}_{sys}$

$$MSE\left(\hat{\theta}_2^{(d)}\right)_{II}-MSE\left(\overline{Z}_{sys}\right)_{II} > 0 \quad (4.15)$$

$$\rho_{yx} > \frac{R^2b_{xz}^2n'^2\overline{Z}^2\theta_2C_z^2\rho_z^*+2\overline{Y}\overline{Z}Rb_{xz}n'\left[-\theta_3\rho_{yz}C_yC_z\sqrt{\rho_y^*\rho_z^*}+\theta_3\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*}\right]}{4\overline{Y}^2\theta_3C_yC_x\sqrt{\rho_y^*\rho_x^*}} \quad (4.16)$$

Estimator $\overline{Z}_{sys}$ is more efficient than $\hat{\theta}_2^{(d)}$ when (4.16) is satisfied.

iv. Comparing Singh *et al*. (2011) exponential ratio/product estimator $\hat{\theta}_3^{(d)}$ with $\overline{Z}_{sys}$

$$MSE\left(\hat{\theta}_3^{(d)}\right)_{II}-MSE\left(\overline{Z}_{sys}\right)_{II} > 0 \quad (4.17)$$

$$\rho_{yx} < \frac{2\bar{Y}\bar{Z}Rb_{xz}n'\left[-\theta_3\rho_{yz}C_yC_z\sqrt{\rho_y^*\rho_z^*}+\theta_3\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*}\right]-\frac{3}{4}\theta_3\bar{Y}^2C_x^2\rho_x^* + R^2b_{xz}^2n'^2\bar{Z}^2\theta_2C_z^2\rho_z^*}{\theta_3\bar{Y}^2C_yC_x\sqrt{\rho_x^*\rho_y^*}} \quad (4.18)$$

Estimator $\bar{Z}_{sys}$ is more efficient than $\hat{\theta}_3^{(d)}$ when (4.18) is satisfied.

v. Comparing Singh *et al*. (2011) exponential ratio/product estimator $\hat{\theta}_4^{(d)}$ with $\bar{Z}_{sys}$

$$MSE\left(\hat{\theta}_4^{(d)}\right)_{II} - MSE\left(\bar{Z}_{sys}\right)_{II} > 0 \quad (4.19)$$

$$\rho_{yx} > \frac{2\bar{Y}\bar{Z}Rb_{xz}n'\left[-\theta_3\rho_{yz}C_yC_z\sqrt{\rho_y^*\rho_z^*}+\theta_3\rho_{xz}C_xC_z\sqrt{\rho_x^*\rho_z^*}\right]-\frac{3}{4}\theta_3\bar{Y}^2C_x^2\rho_x^* + R^2b_{xz}^2n'^2\bar{Z}^2\theta_2C_z^2\rho_z^*}{3\theta_2\bar{Y}^2C_yC_x\sqrt{\rho_x^*\rho_y^*}} \quad (4.20)$$

Estimator $\bar{Z}_{sys}$ is more efficient than $\hat{\theta}_4^{(d)}$ when (4.20) is satisfied.

Empirical study

Five (5) real life data sets were obtained from various sources specified below in order to investigate the efficiency of the two modified estimators.

Data 1: Source (Akkus, 2016)

Y: The amount of produced table olive (tons), X: The number of fruit trees in that age

Z: Collective areas of fruit (decar);

N = 287, n_1 = 165, n_2 = 106; $\bar{Y}$ = 1306.62, $\bar{X}$ = 124081.969, $\bar{Z}$ = 7672.369; C_y = 2.26099, C_x = 3.51001, C_z = 3.51944; P_{xy} = 0.8045, P_{yz} = 0.8188, P_{xz} = 0.965; $\beta_1(x)$ = 9.3154, $\beta_2(x)$ = 110.140, $\beta_1(z)$ = 11.382, $\beta_2(z)$ = 9.3154

Data 2: Source (Singh and Kumar, 2011)

Y: Weight (kg) of the children, X: Skull circumference (cm) of the children, Z: Chest circumference (cm) of the children;

$\bar{Y}$ = 19.4968, $\bar{X}$ = 51.1726, $\bar{Z}$ = 55.1726; C_y = 0.15613, -C_x = 0.03006, C_z = 0.4204; P_{xy} = 0.328, P_{yz} = 0.846, P_{xz} = 0.297; N = 95, n_2 = 24, n_1 = 35

Data 3: Source (Anderson, 1958)

Y: Head length of second son, X: Head length of first son, Z: Head breadth of first son;

N = 25, n_1 = 10, n_2 = 7; $\bar{Y}$ = 183.34, $\bar{X}$ = 185.72, $\bar{Z}$ = 151.12; P_{xy} = 0.7108, P_{yz} = 0.6932, P_{xz} = 0.7346; C_y = 0.0546, C_x = 0.2422, C_z = 0.0488; $\beta_1(z)$ = 0.002, $\beta_2(z)$ = 2.6519

Data 4: Source (Handiquer et al., 2011)

Y: forest timber volume in cubic meter (Cum) in 0.1 ha sample plot, X: average tree height in the sample plot in meter (m) , Z: average crown diameter in the sample plot in meter (m);

N = 2500, n_2 = 25, n_1 = 200; $\bar{Y}$ = 4.63, $\bar{X}$ = 21.09, $\bar{Z}$ = 13.55; P_{xy} = 0.79, P_{xz} = 0.66, P_{yz} = 0.72; C_y = 0.95, C_x = 0.98, C_z = 0.64

Data 5: Source (Khare and Rehman, 2015)

Y: Number of Agricultural labor, X: Area of the Village (hectares), Z: Number of cultivators in the village.

$\bar{Y}$ = 137.9271, $\bar{X}$ = 144.8720, $\bar{Z}$ = 185.188; C_y = 1.3232, C_x = 0.8115, C_z = 1.5521; P_{xy} = 0.773, P_{yz} = 0.786, P_{xz} = 0.819; N = 96, n = 24, n_1 = 60

The Tables (1 – 5) below show the biases, MSEs and PREs of the modified ratio-cum-product estimators $\bar{Z}_{sys}$, $\bar{Z}'_{sys}$ and some existing related estimators under cases I and II using Data Sets 1, 2, 3, 4, & 5. The results shows that all the estimators considered with the exception of sample mean are not unbiased. The results also revealed that the modified estimators $\bar{Z}_{sys}$ and $\bar{Z}'_{sys}$ have minimum MSE and higher PRE compared to other estimators computed in the study. Hence, the method is more efficient and is highly recommended for usage in sample survey.

Table 1: Computation of bias, MSE and PRE of modified estimators $\bar{Z}_{sys}$, $\bar{Z}'_{sys}$ and some related existing estimators using data 1

Estimators	Bias	MSE	PRE
	CASE I		
Sample mean $\bar{y}_{sys}$	0	51926.22	100
Ratio $\hat{\theta}_1^{(d)}$	-8.37421	49340.28	105.241
Singh *et al*. (2011) $\hat{\theta}_3^{(d)}$	-4.074952	32894.73	157.8558
Modified $\bar{Z}_{sys}$	6.14273	32532.09	159.61536
Modified $\bar{Z}'_{sys}$	-5.3101475	32040.76	162.063
	CASE II		
Sample mean $\bar{y}_{sys}$	0	51926.22	100
Ratio $\hat{\theta}_1^{(d)}$	12.4319	34358.29	151.13156
Singh *et al*. (2011) $\hat{\theta}_3^{(d)}$	5.9153	31907.27	162.741
Modified $\bar{Z}_{sys}$	6.14273	29781.86	174.35522
Modified $\bar{Z}'_{sys}$	-4.170977	28058.89	185.06115

Table 2: Computation of bias, MSE and PRE of modified estimators $\bar{Z}_{sys}$, $\bar{Z}'_{sys}$ and some related existing estimators using data 2

Estimators	Bias	MSE	PRE
	CASE I		
Sample mean $\bar{y}_{sys}$	0	0.2885512	100
Ratio $\hat{\theta}_1^{(d)}$	-0.000384497	0.2499512	115.443
Singh *et al*. (2011) $\hat{\theta}_3^{(d)}$	-0.0001705303	0.2538116	113.6872
Modified $\bar{Z}_{sys}$	-0.0003474096	0.2404563	120.0015
Modified $\bar{Z}'_{sys}$	-0.007030953	0.2290472	125.9789
	CASE II		
Sample mean $\bar{y}_{sys}$	0	0.2885512	100
Ratio $\hat{\theta}_1^{(d)}$	0.0003592871	0.2421011	119.1862
Singh *et al*. (2011) $\hat{\theta}_3^{(d)}$	-0.0002711882	0.2470974	116.7763
Modified $\bar{Z}_{sys}$	-0.0003474096	0.2389407	120.7627
Modified $\bar{Z}'_{sys}$	-0.01939473	0.2353535	122.6033

Table 3: Computation of bias, MSE and PRE of modified estimators $\bar{Z}_{sys}$, $\bar{Z}'_{sys}$ and some related existing estimators using data 3

Estimators	Bias	MSE	PRE
	CASE I		
Sample mean $\bar{y}_{sys}$	0	29.55471	100
Ratio $\hat{\theta}_1^{(d)}$	-0.1193633	18.9745	155.7602
Singh *et al*. (2011) $\hat{\theta}_3^{(d)}$	-0.01124068	18.77936	157.3787
Modified $\bar{Z}_{sys}$	0.03520497	17.84369	165.6311
Modified $\bar{Z}'_{sys}$	-7.994106	16.79389	175.9849
	CASE II		
Sample mean $\bar{y}_{sys}$	0	29.55471	100
Ratio $\hat{\theta}_1^{(d)}$	0.1318288	23.51336	125.6933
Singh *et al*. (2011) $\hat{\theta}_3^{(d)}$	-0.005068376	16.72068	176.7555
Modified $\bar{Z}_{sys}$	0.03520497	16.50108	179.10771
Modified $\bar{Z}'_{sys}$	-4.12861	15.52136	190.4131

Table 4: Computation of bias, MSE and PRE of modified estimators $\bar{Z}_{sys}$, $\bar{Z}'_{sys}$ and some related existing estimators using data 4

Estimators	Bias	MSE	PRE
	CASE I		
Sample mean $\bar{y}_{sys}$	0	0.7661334	100
Ratio $\hat{\theta}_1^{(d)}$	-0.1937033	0.3447446	222.2322
Singh *et al*. (2011) $\hat{\theta}_3^{(d)}$	-0.005709978	0.3550018	215.8111
Modified $\bar{Z}_{sys}$	0.03711312	0.3348238	228.8169
Modified $\bar{Z}'_{sys}$	-1.621409	0.2681303	285.7317
	CASE II		
Sample mean $\bar{y}_{sys}$	0	0.7661334	100
Ratio $\hat{\theta}_1^{(d)}$	0.116205	0.3846656	199.1687
Singh *et al*. (2011) $\hat{\theta}_3^{(d)}$	-0.003554418	0.3323451	230.5234
Modified $\bar{Z}_{sys}$	0.03711312	0.3170532	241.6419
Modified $\bar{Z}'_{sys}$	-1.694167	0.2864082	267.497

Table 5: Computation of bias, MSE and PRE of modified estimators $\bar{Z}_{sys}$, $\bar{Z}'_{sys}$ and some related existing estimators using data 5

Estimators	Bias	MSE	PRE
	CASE I		
Sample mean $\bar{y}_{sys}$	0	1040.879	100
Ratio $\hat{\theta}_1^{(d)}$	-3.264956	544.8695	191.0327
Singh *et al*. (2011) $\hat{\theta}_3^{(d)}$	-0.5826492	670.3611	155.2714
Modified $\bar{Z}_{sys}$	-0.4986562	520.3429	200.0371
Modified $\bar{Z}'_{sys}$	-1.002639	516.2941	201.6058
	CASE II		
Sample mean $\bar{y}_{sys}$	0	1040.879	100
Ratio $\hat{\theta}_1^{(d)}$	1.538877	517.3582	201.1912
Singh *et al*. (2011) $\hat{\theta}_3^{(d)}$	-0.6325148	621.371	167.5133
Modified $\bar{Z}_{sys}$	-0.4986562	503.255	206.82947
Modified $\bar{Z}'_{sys}$	-1.259284	455.6917	228.4173

Conflict of Interest

Authors declare that there is no conflict of interest.

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